



Viewing the world systemically.

Developing an *ATIS*-Topology

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Developing an ATIS Topology

In its most general form, *topology* is concerned with how things are connected.

While it is frequently thought of in terms of geometrical forms, it is important to avoid this confusion.

[Geometry](#) is concerned with describing the shapes of things.

[Topology](#) is concerned with connectedness.

Thought of in this way, it helps to eliminate that confusion, and suggests applications not generally considered as being topological.

Stephen Barr, [Experiments in Topology](#),¹ gives the following examples of topology applications:

It frequently happens that when getting a cup of coffee one forgets the cream. The trick, here, is not to go and get the cream, but to take the cup to it. The first way involves four trips: going for the cream, bringing it to the table, taking it back right away, and returning to the coffee. The other way involves two: taking the cup to the refrigerator and returning with the cup. This cannot be helpfully expressed geometrically, but the kind of sequential planning used, though arithmetical, belongs rather in topology. (p. 197)

That is, the problem is concerned with *connectedness*. And, topology is used frequently in everyday life:

Most descriptions of an objects location are topological, rather than geometrical: The coat is in your closet; the school is the fourth house beyond the intersection of this street and Route 32; The Pen of my Aunt is in the Garden.

Again, these problems are concerned with *connectedness*. The value of topology to behavioral theorizing is seen in the importance of the multitude of components in a behavioral system that are connected, and the importance of the kinds of connectedness.

¹ Barr, Stephen, [Experiments in Topology](#), Dover Publications, 1989.

Topology and Intentional Systems Theory

The value of theory in general, and behavioral theory in particular, is that theory provides a means of **predicting outcomes**. To date, behavioral sciences have had to rely on empirical testing to arrive at predictive assertions. That is, given a hypothesis, experiments must be conducted in order to *verify* the hypothesis.

The difficulty with all such testing and any conclusions derived therefrom is that they are dependent upon statistical measures that are only **group-predictive**, and not **individually-predictive**. A further and perhaps far more important difficulty is that when considering hypotheses, there is no assurance that different hypotheses actually have the same basic assumptions; and, in fact, they probably do not. Without the same basic assumptions for two different hypotheses, they cannot be incorporated into the same theory. This problem persists even for hypotheses that are designed to study the same or similar types of events. In fact, many times hypotheses are revisited in order to refute the findings of one as opposed to another by claiming that the very foundations of the hypotheses compromised the study.

An axiomatic-based system description is critical for an intentional, behaviorally-predictive system. Predictions derived therefrom are not dependent on the result of previous behaviors, experiments or outcomes. **Predictions are dependent on a parametric analysis of an existing system state.**² A sequence of previous system states can define a dispositional system behavior, but are used, not as a definitive guideline for predicting future behavior, but as part of a comprehensive analysis of the existing system state.

By analyzing the *structure* of the behavioral (intentional) system, conclusions; that is, **predictions** can be obtained from a **parametric analysis** of the system.

Parametric analysis is the analyzing of hypotheses of a theory based only upon its parameters.

² A *parametric analysis* is an analysis of relationships between system components. A *nonparametric analysis* is an analysis of relationships between descriptive; that is, non-specific, and inferred relationships that a researcher may propose in the process of identifying system components in a *rough set*. *Classical sets* contain elements (components) that are well-defined, and elements can be specifically determined as to whether or not they belong to the sets. *Fuzzy sets* contain elements (components) that are not well-defined or are vaguely defined so that it is indeterminate which elements (components) belong to the sets although other elements (components) may be well-defined as in *classical sets*. *Rough sets* are defined by topological approximations, which include elements (components) that are well-defined as in *classical sets*, and elements (components) that may or may not be in the set. These potentially *rough set* components are not *fuzzy set* elements (components) since they are not vaguely defined, they are just unknown concerning the set property.

An added value to this type of analysis is that predictions relating to intentional systems can be made from their *nonempirical structural parameters*.³ In fact, this is the only feasible way to ever analyze an intentional system with any assurance of the reliability of any outcomes. The reason is due to the very large number of structures contained in even the smallest intentional system. *ATIS* generates thousands of theorems which, when applied to specific intentional systems, will result in millions of possible *hypotheses* (that is, *theorems*) being generated.

Analyzing these systems by means of a *parametric analysis of their nonempirical structural parameters* appears to be the only reliable avenue to ever achieving the predictive results desired.

Further, it then becomes possible to evaluate a particular intentional system by first evaluating a formal system that is homeomorphic to the behavioral system.⁴ Any topological invariants will be the same for both systems, thus eliminating the necessity of conducting empirical tests for each and every distinct behavioral system. If they are homeomorphic, predictions can be made from the formal system about the empirical system.

³ *Nonempirical Structural Parameters*, NeSPs, are discussed in a separate report—*QSARs, QSPRs, and their relevance to ATIS*. It is intended that this report will soon be published at some time in 2016.

⁴ See [ATIS Properties: Morphisms](#).

MATHEMATICAL TOPOLOGY & BEHAVIORAL (Intentional Systems) THEORY

In this study, we will not be considering the study of topology, but will be considering the application of topology to behavioral theory. The applicability of topology to behavioral theory is suggested by [Klaus Jänich](#) in his text *Topology*⁵ at page 2:

“... the application of point-set topology to everyday uses in other fields is based not so much on deep theorems as on the unifying and simplifying power of its system of notions and of its felicitous terminology.”

It is this “unifying and simplifying power” that we will develop for *Intentional Systems Theory*⁶. Following this development, the full power of mathematical topology in terms of its analytic tools and theorems will depend upon the intentional systems involved and the creativity of the analyst. Specific applications will be suggested throughout this development, and a constructive approach to analyzing behavioral topological spaces will be presented. Various definitions and theorems will be presented just to indicate what topological analyses may be of value in analyzing specific intentional systems.

Topologies and Topological Spaces

There are a number of ways that a [topological space](#) can be defined. The one chosen herein was selected due to its potential for properly evaluating the desired concepts of *Intentional Systems Theory*.

[Topology](#) is the study of those properties of a system that endure when the system is subjected to topological transformations. This introduces the initial rationale for interpreting the structural properties as a topology—to be able to analyze properties of a system in a manner that can distinguish between substantive and non-substantive distinctions. Having initiated this interpretation, however, it may be seen that other interesting and beneficial evaluations will arise.

A **topological transformation** is a continuous transformation that has a continuous inverse transformation. That is, a **topological transformation** is a continuous transformation that can be continuously reversed or undone. Two systems are **topologically equivalent** if there is a topological transformation between them.

⁵ Jänich, Klaus, *Topology*, Springer, 1995.

⁶ For a discussion of the focus of *Intentional Systems Theory*, see [Intentional Systems](#) and [Dynamic Teleological System](#).

Thus, the tools are available to determine whether or not two intentional systems are in fact substantively different, or if they only appear to be different due to our vision being determined by geometric perspectives. That is, things are normally differentiated “geometrically,” whereas differentiating them topologically will get at their substantive distinctions. That is, normally distinctions may be “seen” where in fact there are none.

A **topological property**; i.e., **topological invariant**, of a system is a property possessed alike by the system and all its topological equivalents. A **topological invariant** always carries information concerning one or more topological properties.

Topological space and topology are defined as follows.

Definition: *Topological Space and Topology.*

$T = (\mathfrak{S}_x, \tau)$ is a **topological space**, where \mathfrak{S}_x is a **set of points** and τ , the **topology**, is a class of subsets of \mathfrak{S}_x , called **open-neighborhoods**, such that:

- (1) Every point of \mathfrak{S}_x is in some open-neighborhood, \mathcal{N}_i , $i \in \mathbb{I}^+$, the set of positive integers;
- (2) The intersection of any two open-neighborhoods of a point contains an open-neighborhood of that point; and
- (3) \mathfrak{S}_x and \emptyset are elements of τ .

Formally:

$$\tau = \{ \mathcal{N}_i \mid \mathcal{N}_i \subseteq \mathfrak{S}_x \wedge i \in \mathbb{I}^+ \wedge \forall x \in \mathfrak{S}_x \exists \mathcal{N}_i (x \in \mathcal{N}_i \wedge (x \in \mathcal{N}_j \wedge x \in \mathcal{N}_k \supset \exists \mathcal{N}_i (x \in \mathcal{N}_i \subset \mathcal{N}_j \cap \mathcal{N}_k)) \cup \{ \mathfrak{S}_x, \emptyset \}.$$

That is, τ is the **topology** that consists of **open-neighborhood sets**, \mathcal{N}_i , of \mathfrak{S}_x such that every point of \mathfrak{S}_x is in some \mathcal{N}_i , and the intersection of any two neighborhoods of a point of \mathfrak{S}_x contains a neighborhood of that point, and \mathfrak{S}_x and \emptyset are elements of τ .

Properties of Topological Spaces for an *Intentional Systems Theory*

It is proposed that the following properties of topological spaces will be useful in analyzing intentional systems. Where appropriate, especially for vector fields, these properties are defined specifically for *Intentional Systems Theory*.

Definition. *Open Sets.*

T is a topological space, (\mathfrak{S}_x, τ) . The elements of τ are **open sets**.

Definition. *Neighborhood of a Point.*

The set, \mathcal{N} , is a neighborhood of a point, p , if: (1) $p \in \mathcal{N}$; and (2) \mathcal{N} is open.

Definition. *Near (Point-to-Set).*

T is a topological space, (\mathfrak{S}_x, τ) . Let $S \subset \mathfrak{S}_x$ and $\mathfrak{x} \in S$. \mathfrak{x} is **near** S , $\mathfrak{x} \leftarrow S$, if every neighborhood of \mathfrak{x} contains an element of S .

Definition. Let $T = (\mathfrak{S}_x, \tau)$. Let $S \subset \mathfrak{S}_x$. Then:

Path.

Any set topologically equivalent to the line segment $[0,1]$.

Closed Path or Jordan Curve.

Any set topologically equivalent to the circle, $[P] = 1$.

Closed.

S is **closed** if it contains all its near points.

That is, there are no neighborhoods other than S that contain the elements of S .

Open.

S is **open** if every element in S is not near the complement of S , S' .

That is, there are no neighborhoods for the elements of A which contain elements of S' .

Clopen.

S is **clopen** if it is both open and closed. $[\mathfrak{S}_x$ and \emptyset are always clopen in a topology.]

Connected.

S is **connected** if for every nonempty disjoint partition of S , U and V , one partition contains an element near the other.

Theorem: *Paths are connected.*

Following are additional topological properties that will be of value in analyzing various intentional systems.

Definitions: Let $T = (\mathfrak{S}_x, \tau)$. Let $S \subset \mathfrak{S}_x$. Then:

Interior Point.

$x \in \mathfrak{S}_x$ is an **interior point** of S if S is a neighborhood of x .

Exterior Point.

$x \in \mathfrak{S}_x$ is an **exterior point** of S if S' is a neighborhood of x .

Boundary Point.

$x \in \mathfrak{S}_x$ is a **boundary point** of S if neither S nor S' is a neighborhood of x .

Set Interior.

If S^o is the set of interior points of S , then S^o is the **interior** of S .

Set Closure.

If S^- is the set of points of S which are not exterior points, then S^- is the **closure** of S .

That is, S^- is the set of interior and boundary points of S .

Definition: *Disjoint Union (Sum) of Sets.*

Let $X + Y = X \times \{0\} \cup Y \times \{1\}$, then $X + Y$ is the **disjoint union** or **sum** of X and Y .

By this definition we obtain a copy of X and Y individually, rather than their union.

With the following definition, we can obtain a new topology from two given topologies as follows.

Definition: *Disjoint Union (Sum) of Topological Spaces.*

If (X, τ) and (Y, τ^*) are topological spaces, then a new topology on $X + Y, \tau^{**}$, is given by:

$$\tau^{**} = \{U + V | U \in \tau \text{ and } V \in \tau^*\},$$

and $(X + Y, \tau^{**})$ is called the topological disjoint union of the topological spaces X and Y .

This may prove fruitful when considering different topologies within the same behavioral system, or when trying to join two different systems.

We will now consider the definition of a **product topology**. The generalization of this definition will require the following definition of **product set**.

Definition: *Product Set.*

Let $\{X_i | i \in I^+\}$ be a family of sets.

[**NOTE:** I^+ may be replaced with a finite index set, and will generally be done so with a particular intentional system.]

Then,

$$X_I + X_i = \{(x_1, x_2, \dots, x_i, \dots) | \forall i \in I^+ (x_i \in X_i)\}$$

If $x = (x_1, x_2, \dots, x_i, \dots) \in X_I + X_i$, then,

x_i is called the **i-th coordinate** of x ;

X_i is called the **i-th component** of $X_I + X_i$; and

$X_I + X_i$ is called the **product set** of the sets $\{X_i | i \in I^+\}$

Definition: *Product Topology.*

If (X, τ) and (Y, τ^*) are topological spaces, then τ^{**} is the **product topology** of $X \times Y$ and is given by:

$$\tau^{**} = \{(x,y) | \exists \mathcal{N}(x) \exists \mathcal{N}(y) (x \in X \wedge y \in Y \supset \mathcal{N}(x) \in \tau \wedge \mathcal{N}(y) \in \tau^*)\}.$$

Generalizing this definition, we have the following:

Let $\{(X_i, \tau_i) | i \in I^+\}$ be a countable family of topological spaces. Then,

$$\tau = \{X_I + X_i | i \in I^+ \wedge \forall x_i \in X_i \exists \mathcal{N}(x_i) (x_i \in X_i \Rightarrow \mathcal{N}(x_i) \in \tau_i)\}$$

is the **product topology** of $X_I + X_i$.

Definition: *Continuous.*

S and T are topological spaces. A transformation, $f: S \rightarrow T$, is **continuous** if for any point, $x \in S$, and subset, $A \subset S$, $x \leftarrow A \Rightarrow f(x) \leftarrow f(A)$.

Definition: *Bijjective Map.*

$f: X \rightarrow Y$, is a **bijjective map** if f is 1-1 and onto.

Definition: *Homeomorphism.*

$f: X \rightarrow Y$, is a **homeomorphism** if f is bijective and continuous, and f^{-1} is continuous.

The following connectedness theorems may prove useful when analyzing various intentional systems.

Theorem: *Continuous Images Maintain Connectedness.*

If X is a (path-)connected space and $f: X \rightarrow Y$ is continuous, then $f(X)$ of Y is also a (path-)connected space.

Theorem: *Non-Disjoint Unions Maintain Connectedness.*

If X_0 and X_1 are (path-)connected subspaces of X , $X = X_0 \cup X_1$, and $X_0 \cap X_1 \neq \emptyset$, then X is (path-)connected.

Theorem: *Products Maintain Connectedness.*

$X_1 \times X_i$ of non-empty topological spaces is (path-)connected if and only if all X_i are.

Topological Vector Fields

For the following definitions: $T = (\mathcal{S}, \tau)$ is a topological space and $P, Q \subset \mathcal{S}$.

Definition: *Direct Affect Relation.*

\mathbf{A} is a **direct affect relation** from x to y defined by $\mathbf{A}: P \rightarrow Q$, where $x \in P$ and $y \in Q$, such that $\{x, y\} \in \tau$, and $\mathbf{A}(x) = y$ is defined for all x of P .

Definition: *Direct Affect Relation Measure.*

The function, M , defined by $M: P \times Q \rightarrow \mathcal{R}$, is a **direct affect relation measure** of the direct affect relation, \mathbf{A} , defined by $\mathbf{A}(x) = y$, such that $M(x, y) = m$.

Definition: *Vector.*

v is a **vector** from x to y , $x \rightarrow y$, if there is a direct affect relation, \mathbf{A} , and a direct affect relation measure, M , defined for $x \in P$ and $y \in Q$.

Definition: *Vector Field.*

V is a **vector field**, if $V = \{(x, y) | x \in P, y \in Q, x \rightarrow y\}$.

The value of *APT* methodologies now becomes quite apparent. The intentional systems theory provides the parametric formulas for analyzing an intentional system. *APT* provides the means for obtaining the information required to apply these analyses to specific intentional systems.

In particular, *APT* makes it possible to evaluate the family of **Affect Relation Vector Fields**. That is, any behavioral system will have numerous vector fields, which normally would have to be analyzed individually. *APT* provides the means to evaluate them simultaneously. That is, they are viewed in terms of the *APT Map*. This should prove to be most beneficial when applying the theory to a particular individual or system. *APT Maps* provide the methodologies to make the theory individually predictive or system-specific predictive.

Further, the theory, in terms of either the disjoint union or product of topological spaces will provide the theoretical perspective required to assign appropriate parameters to the *APT Map*, and then to analyze the results of that score in terms of their theoretical significance.

Definition: *Fixed Point and Fixed Point Property.*

Let f be a continuous transformation from X into X , represented by:

$$f_1: X \rightarrow X, f_2: X \rightarrow X, f_3: X \rightarrow X, \dots, f_i: X \rightarrow X, \dots, f_n: X \rightarrow X.$$

Then, if there is an $x \in X$ such that $f_1(x) = f_n(x) = x$, then x is called a **fixed point** of f . If for every f , X has a fixed point, then X has the **fixed-point property**.

Theorem: *Brouwer's Fixed Point Theorem.*

Cells have the fixed-point property.

Definition: *Self-Similar.*

A system that is homeomorphic to any subset of the system.

Constructive Development of a Topology for an Intentional System

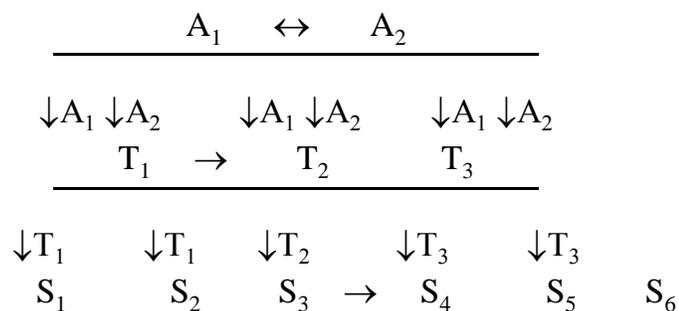
In order to consistently analyze intentional systems, a “Construction Rule” will be developed that will generate a topology in the same manner for every system analyzed.

Topology Construction Rule:

- (1) Every element of the system is contained in a neighborhood consisting of one element, and are designated as the family of null-affect neighborhoods, A_0 .
- (2) Every element of the system is classified by type and is assigned to the neighborhood containing just elements so classified. All such neighborhoods are designated as the family of descriptive neighborhoods, D_0 .
- (3) All elements with affect relations between them are pair-wise assigned to a neighborhood containing just those two elements, and are designated as a family of affect relations by type; that is, A_1, A_2, A_3 , etc.
- (4) The system and the null set are elements of the topology.

Now, let’s see how this works.

Let $S = \{A_1, A_2, T_1, T_2, T_3, S_1, S_2, S_3, S_4, S_5, S_6\}$; where “A” represents “Armed Forces Personnel,” “T” represents “Terrorist Groups,” and “S” represents “Sites Targeted.” Graphically define the following affect relations shown by the arrows.



By the above *Topology Construction Rule*, this generates the following neighborhoods for the topology:

$$A_0 = \{\{A_1\}, \{A_2\}, \{T_1\}, \{T_2\}, \{T_3\}, \{S_1\}, \{S_2\}, \{S_3\}, \{S_4\}, \{S_5\}, \{S_6\}\}$$

$$D_0 = \{\{A_1, A_2\}, \{T_1, T_2, T_3\}, \{S_1, S_2, S_3, S_4, S_5, S_6\}\}$$

$$A_1 = \{\{A_1, A_2\}, \{A_1, T_1\}, \{A_1, T_2\}, \{A_1, T_3\}, \{A_2, T_1\}, \{A_2, T_2\}, \{A_2, T_3\}\}$$

$$A_2 = \{\{T_1, T_2\}\}; \quad A_3 = \{\{T_1, S_1\}, \{T_1, S_2\}, \{T_2, S_3\}, \{T_3, S_4\}, \{T_3, S_5\}\}; \quad A_4 = \{\{S_3, S_4\}\}$$

Thus,

$$\tau = A_0 \cup D_0 \cup A_1 \cup A_2 \cup A_3 \cup A_4 \cup \{S, \emptyset\}.$$

This topology can be easily verified. And, the construction can be easily determined to always result in a topology, since every intersection will result in either a set of one element, which is in the topology, or in a set of two elements that is in the topology.

Now, let's see what may be determined that may not otherwise be too obvious. Granted, given this limited intentional system, this result may be somewhat obvious, but it does indicate the power of this analysis given much larger systems.

Are (T_1, S_4) and (T_1, S_5) path-connected?

The following sequence of neighborhoods determines that both pairs are path-connected.

For (T_1, S_4) : $\{T_1, T_2\}, \{T_2, S_3\}, \{S_3, S_4\}$.

For (T_1, S_5) : $\{A_1, T_1\}, \{A_1, A_2\}, \{A_2, T_3\}, \{T_3, S_5\}$.

While, intuitively, it may seem that (T_1, S_5) are not path-connected due to the direction of the arrows; that is, the affect relations, an analysis of the system indicates that they are so connected. This should not be too surprising, since the impact of the "system" concept is that elements of the system are in fact responsive to changes in or influences by other elements of the system. This does not mean that "vectored direct affect relations" cannot be determined; however, the question here is, do affects by T_1 influence, or have an effect on, S_5 ? The answer is "Yes." That degree of influence is not yet determined.

Now, another result of the above example is to see that an element may be connected but not path-connected. For example, S_6 is connected but not path-connected. As seen from the example, there are no affect relations connecting S_6 to the other elements. In the topology, there are only two neighborhoods that contain this element; that is, $\{S_1, S_2, S_3, S_4, S_5, S_6\}$ and $\{S_6\}$. They, clearly, are not path-connected. But, are they connected? Yes. They are connected by the subset $\{\{S_1, S_2, S_3, S_4, S_5, S_6\}, \{S_6\}\}$. That is, when divided into the only two non-empty disjoint parts, then one part contains a point near the other, specifically S_6 .

This is certainly a desired result in an intentional system. Since, obviously, a new *Site Target* can be introduced into the system at any time. Yet, at the time of introduction, there may not be any substantive affect relation established. The topological space must allow for that relation to be established. Of course, at the time of introduction into the system, certain affect relations may be established, but, not necessarily those under consideration. Clearly, a behavioral topology can become extremely complex. That complexity is being minimized for the sake of introducing topology as a tool for eventual analysis of that system's complexity.

Now for a consideration of the *vectorized direct affect relations*.

The following concepts are derived from Michael Henle's *A Combinatorial Introduction to Topology*.⁷

A vector field V on a subset D of the plane is a function assigning to each point, P of D , a vector in the plane with its initial point at P . "*Intuitively, we can think of V as giving the velocity of some substance that is presently in D* " (p. 33).

That is, we can think of affect relations as the "substance" being influenced, and the vector, V , as that which is giving those relations "velocity" or "impact." The vector is that which makes the affect relations effective.

The essential qualities of a vector are its length and direction. For our purposes, "length" can be defined as the "power" or "force" or "degree" of influence or effectiveness of the affect relation. "Direction" can be defined as the recipient of the affect.

The importance of vectors for *Intentional Systems Theory* can be seen from the following description provided by Henle:

⁷ Henle, Michael, *A Combinatorial Introduction to Topology*, Dover Publications, Inc., New York, 1979.

“Clearly the study of vector fields on a set D coincides with the study of continuous transformations of the set [that is, topology].

Vector fields have many important applications. The force fields arising from gravitation and electromagnetism are vector fields; the velocity vectors of a fluid in motion, such as the atmosphere (wind vectors), form a vector field; and gradients, such as the pressure gradient on a weather map or the height gradient on a relief chart, are vector fields. These examples are usually studied from the point of view of differential equations. A vector field, $V(P) = (F(x,y), G(x,y))$, determines a system of differential equations in the two unknowns x and y . These variables are taken to represent the position of a moving point in the plane dependent on a third variable, the time t . The system of differential equations takes the form: $x' = F(x,y)$, $y' = G(x,y)$; where the differentiation is with respect to t . Such a system is called **autonomous** because the right-hand sides are independent of time. A solution of this system consists of two functions expressing x and y in terms of t . These may be considered the parametric equations of a path in the plane: the path of a molecule of gas or liquid, the orbit of a planet or an electron, or the trajectory of a marble rolling down a hill, depending on the application. The original vector field $V(P)$ gives the tangent vector to the path of motion at the point $P = (x,y)$.”

The “application” here is with respect to the affect relations of an intentional system. Such an application will be required if the full impact of topology is to be realized in this study. And, it will clearly be required in order to introduce the time element that is critical to any in-depth analysis of an intentional system.