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The SimEd Basic Logic as Founded on the Logic of Axiomatic-General Systems Behavioral Theory:

Properties: Morphisms

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Properties: Morphisms

Introduction

Morphism, $\mathcal{M}$, is a “map” from one “object” to another “object.” Objects and morphisms comprise a category. That is, a ‘category’ is a collection of ‘objects’ such that for any two objects there is a collection of ‘mappings’. This is a mathematical definition of ‘category’. In fact, it is very similar to the definition of ‘system’; that is, a system is comprised of a set and relations defined on the set.

Due to this similarity of definition, morphisms can be used to define the various types of affect-relations where the ‘objects’ are the systems and subsystems being considered.

Automorphism, $\mathcal{A}$, = def components of the same system whose connections are transformed so that the same connections are maintained.

$$\mathcal{A} = \text{def} \quad \mathcal{M}(S_1, S_1) \mid I(S_1, S_1)$$

Automorphism is defined as an isomorphism of a system onto itself.

Commensalmorphism, $\mathcal{C}$, = def a relation between systems by which one maintains steadiness while the other increases complexity.

$$\mathcal{C} = \text{def} \quad \mathcal{M}(S_1, S_2) \mid S_1(\mathcal{X}) \land S_2(\mathcal{X})$$

Commensalmorphism is defined as a mapping between two systems; such that, the first system maintains a steady state while the second has complexity-growth.
**Endomorphism, \( \varphi \), = \text{df} \text{ System components whose connections are transformed so that a different system state is obtained while maintaining the same components.}**

\[
\varphi = \text{df} \mathcal{M}(\mathcal{S}_1:1_{t(1)},\mathcal{S}_1:1_{t(2)}) \mid \sigma(\mathcal{S}_1:1_{t(1)}(\mathcal{S}),\mathcal{S}_1:1_{t(2)}(\mathcal{S})) \Rightarrow \neg(\mathcal{S}_1:1_{t(1)}(\mathcal{S}) \equiv \mathcal{S}_1:1_{t(2)}(\mathcal{S})),
\]

**Endomorphism** is defined as a homomorphism of a system onto itself at two different times; such that, there is a state-transition between time 1 and 2, implies that the two system states are not equivalent (that is, the system state has changed). An endomorphism results when the component path-connections of a system are mapped onto dissimilar path-connections; that is, the same system components are maintained but are rearranged so that new path-connections are obtained. The result of an endomorphism is a rearrangement of components that does alter the system state while maintaining the same components.

**Epimorphism, \( \varphi \), = \text{df} \text{ components of two disjoint systems in which the components of one are related to every component of the other.}**

\[
\varphi = \text{df} \mathcal{M}(\mathcal{S}_1,\mathcal{S}_2) \mid \\
\mathcal{S}_1 \cap \mathcal{S}_2 = \emptyset \land \forall y \in \mathcal{S}_2(\mathcal{M}(\mathcal{S}_1,\mathcal{S}_2)) \land \exists y \in \mathcal{S}_2 \exists a,b \in \mathcal{S}_1(\mathcal{M}(\mathcal{S}_a,\mathcal{S}_2) \land \mathcal{M}(\mathcal{S}_b,\mathcal{S}_2))
\]

**Epimorphism** is defined as a homomorphism; such that, the two systems are disjoint, and for every component of the second system there is a component of the first system mapped to it, and there is a component of the second system that has two components of the first system mapped to it. An epimorphism is an “onto” but not “one-to-one” mapping.

**Homeomorphism, \( \mathcal{H} \), = \text{df} \text{ corresponding components of two disjoint topological spaces have the same connections.}**

\[
\mathcal{H} = \text{df} f:W^{o} \rightarrow Z \mid W^{o} = (\mathcal{S}_{O(1)},\mathcal{T}_1) \land Z = (\mathcal{S}_{O(2)},\mathcal{T}_2) \Rightarrow \mathcal{L}(W^{o},Z)
\]

**Homeomorphism** is defined as a topological mapping; such that, the spaces are isomorphic.

**Homomorphism (general morphism), \( \mathcal{M} \), = \text{df} \text{ components that have the same connections as other components.}**

\[
\mathcal{M} = \text{df} \mathcal{M}(\mathcal{S}_1,\mathcal{S}_2) \mid f[P(\mathcal{A}_1) \equiv P(\mathcal{A}_2)]
\]

**Homomorphism** is defined as a morphism; such that, the mapping is defined by equivalent affect-relation set predicates of each system.

Every affect relation of a system defines a homomorphism that is self-mapped. Between two systems, a homomorphism is defined by affect-relation sets that are defined by the same predicate.
**Isomorphism**, \( I =_{df} \) corresponding components of two systems that have the same connections.

\[
I =_{df} \mathcal{M}(S_1, S_2) \mid \mathcal{M}(S_1, S_2) = \mathcal{M}(S_2, S_1)
\]

**Isomorphism** is defined as a homomorphism; such that, there is a homomorphism from \( S_1 \) to \( S_2 \) and from \( S_2 \) to \( S_1 \). An **isomorphism** is a bijective homomorphism.

**Monomorphism**, \( N =_{df} \) two systems where the components of one are related to one and only one component of the other, but are not related to all components of the second.

\[
N =_{df} \mathcal{M}(S_1, S_2) \mid \neg \mathcal{P}(S_1, S_2) \land \forall x \in S_1 \Rightarrow \exists y \in S_2(\mathcal{M}(x, y))
\]

**Monomorphism** is defined as a homomorphism; such that, the mapping is not epimorphic (onto), and for every component of the first system there is exactly one component of the second onto which it is mapped (that is, it is one-to-one).

**Morphism**, \( \mathcal{M}^\circ =_{df} \) Two systems whose components are affect-related.

\[
\mathcal{M}^\circ =_{df} f : S_1 \rightarrow S_2 = \mathcal{M}^\circ(S_1, S_2) \mid S_1 = (S_{0(1)}, A_1) \land S_2 = (S_{0(2)}, A_2) \land \exists x \in S_1 \Rightarrow \exists y \in S_2(f(x) = y)
\]

**Morphism** is defined as a mapping of one system into another system; such that, there is a component of the first system implies that there is a component of the second system such that the two components are affect-related.

**Symbiomorphism**, \( \mathcal{S} =_{df} \) a relation between systems that produces state steadiness in both systems.

\[
\mathcal{S} =_{df} \mathcal{M}(S_1, S_2) \mid \mathcal{M}(S_1, S_2) \Rightarrow S_1(S) \land S_2(S)
\]

**Symbiomorphism** is defined as a homomorphism between two systems; such that, the homomorphism implies that both systems obtain steady state.